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Precise Tracking Trhough the Uncertain Atmosphere

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Summary

Bias errors caused by tropospheric refraction may now be corrected to a level below the threshold of the best radars. In most cases, only surface weather data is needed for this. The errors which remain will fluctuate at frequencies from one cps down to one cycle per day, or less, and will lead to serious errors in both radar and interferometer systems. Long baselines reduce the error, but the presence of long-period components in both space and time causes the error to drop very slowly as the baseline is extended. Effects of these components on position and velocity data are shown. Although ionospheric errors can be held to the required level by using frequencies above 3000 mc, it is concluded that precise tracking systems must use trilateration over very long baselines to achieve the accuracies now desired.

Introduction

Instruments available for tracking of aircraft and missiles during the 1940's were limited in accuracy primarily by internal defects rather than by the atmosphere. As development proceeded toward instruments of greater accuracy, the problems of atmospheric refraction were given intensive study, and procedures were devised for applying partial corrections for range and angle errors introduced by the troposphere. Progress in radio meteorology was rapid enough to keep the residual errors of the troposphere below the level of instrumental error until the late 1950's, when several types of microwave tracking instrument reached the limits of atmospheric uncertainty. Throughout this period, electronic systems operating in or below the VHF band were encountering the ionospheric limits to accuracy which still prevent the exploitation of this portion of the spectrum for precise tracking of high-altitude targets.

We will review here the ways in which the atmosphere introduces error in various tracking systems, and some of the important studies which have provided partial corrections for these errors. Tropospheric problems will be taken first, following which we will make a brief survey of ionospheric effects before summarizing the influence of the entire atmosphere on tracking systems.

Tropospheric Range and Angle Bias

The average refractive effects of a spherically stratified atmosphere on radio tracking instruments have been computed and presented in several ways, using slightly different approximations of the "standard atmosphere".[1-4] To illustrate the magnitude of effects to be expected, we have plotted in Figures 1 and 2 the bias errors in range and elevation angle for the CRPL Exponential Reference Atmosphere which closely matches the world-wide average for sea level stations.[3] The errors are dependent upon target range and altitude, running from several feet to about 300 feet in range, and from several tenths of one milliradian to several mr in elevation angle. Even early tracking radars such as the SCR584 required corrections in order to stay within their error specifications at long range. For aircraft tracking, corrections were applied in range by setting the basic ranging oscillator to a frequency somewhat below that which would have corresponded to the vacuum velocity of light. Computations made on the basis of an assumed earth's radius equal to 4/3 times its actual value could be used to remove some of the error in elevation data. If greater accuracy were required, the instrumentation user could select from the literature an atmospheric model and a correction procedure which met his average site conditions, target flight range, and accuracy requirements, and expect to reduce the "bias" errors to a point below the measurement threshold of his system. For tracking with radars derived from the SCR584, this level was about 20 feet or 0.3 mil rms, thus calling for removal of, at most, 90% of the initial error.

Correction of Bias Variations

Bias error corrections based on the standard or local average atmosphere were not of sufficient accuracy to meet the needs of the 1950's. Variations in refractive index profiles, from place to place and over periods of hours to months, caused substantial changes in range and elevation refractive error, and also introduced some error in azimuth measurements. Two general approaches were used to reduce these errors: measurement of spatial and temporal variations in refractivity to improve the correction procedures, and development of new instrumentation systems which exhibited reduced sensitivity to the fluctuations.

The first step in refinement of refraction corrections involved the use of surface measurements to adjust the scale of the refractivity profile. Considerable success was achieved in predicting the total atmospheric bending and range error (as would apply to targets well above the atmosphere), using measurements taken on the ground.[5,6] This procedure was also extended to targets within the atmosphere, leading to correction procedures which were adequate for use with a radar of the AN/FPS-16 class over most of its operating region.[7,8] For instance, the following simple formulae gave results which permitted the achievement of system accuracies near 10 feet rms in range and 0.1 mil rms in angle for targets above 6^{o}:

$$\triangle R_e = 0.0235 \frac{N_s}{\sin E_o} \times \frac{h + 0.1 h^2}{50 + h + 0.1 h^2} \qquad (1)$$

$$\triangle E_o = \frac{N_s}{\tan E_o} \times \frac{h}{35 + h} \qquad (2)$$

Here, $\triangle R_e$ is the range error in feet, N_s the surface refractivity in N-units (or parts per million of the refractive index), E_o is the elevation angle, h is the target altitude in kilofeet, and $\triangle E_o$ is the elevation error in microradians.

Although the accuracy could be improved further by using more complex computation procedures, significant gain in accuracy for the case of targets within the atmosphere is dependent upon use of actual measured profiles of refractivity. An estimate of the optimum correction accuracy for radar data, based on the average of several expert (and conflicting) viewpoints, was given by a committee of the National Academy of Sciences last year, and is shown in Table I below.[9]

Sensitivity of Instrumentation Systems to Refraction

Development of instruments which would have reduced sensitivity to atmospheric (as well as instrumental) sources of error was carried out during the 1950's, leading to the so-called baseline type of tracking system. From the standpoint of refraction error, we obtain two advantages by performing measurements over a long, horizontal baseline. It can be shown that the first-order elevation bias error, in the long-range case, is absent from this system.[9,10] It is therefore unnecessary to measure N_s in order to correct the elevation bias error at high elevation angles, since the term ($N_s/\tan E_o$) from Eq. (2) is applied by the geometry of the instrument. A second advantage lies in the ability of long-baseline systems to reduce the high-frequency components of fluctuating error, as shown in Figure 3.

This figure emphasizes the common basis for measurement of target position in all radio tracking systems: the comparison of phase or time delay at different points on the received wave front. The pulse radar, Figure 3a, measures the round-trip time delay of the envelope of the transmission to obtain range, and adjusts the plane of its antenna so that it lies parallel to the phase front of the received r-f signal, averaged over the antenna aperture. (The pulse Doppler radar also uses the r-f phase information to refine its range precision). The interferometer, Figure 3b, samples the received phase fronts at two or more separate points, and computes the angle of arrival from knowledge of wavelength and system geometry. Ambiguities in the measurement are resolved by using multiple antennas with different spacing. The effect of phase fluctuations which produce one or more cycles of "ripple" in the wave front between the antennas is reduced, as compared to the smaller tracking antenna, although a continuous antenna of equal length would do an even more effective smoothing job if it could be used. Phase fluctuations whose wavelength is longer than the interferometer baseline will produce the same error in either case.

A third type of system, Figure 3c, extends the baseline beyond the point where r-f phase ambiguities can be resolved, and locates the target by "trilateration", comparing the envelope delay (or phase difference of the modulation waveform) as observed at the two sites. From the point of view of tropospheric refraction, it makes no difference whether the carrier or the envelope delay is measured, and the significant difference between systems is the maximum extent of the baseline and the number of samples (if any) which are gathered between the end points. Angle-tracking antennas are limited to about 200 feet in diameter, for economic reasons, and generally are in the 10 to 100 foot class. Interferometers have been built to use spacings of 100 to 100,000 feet, although resolution of phase ambiguities is not generally possible beyond a few thousand feet in microwave systems. Trilateration systems may be extended to approach the diameter of the earth, and further extension to satellites is not impossible if earth-based systems prove inadequate.

When the interferometer systems were first developed, it was believed that fluctuations in tropospheric refraction were, for the most part, limited in size, and that extension of baseline length beyond this size would result in errors almost inversely proportional to baseline length.[11] According to this theory, an Azusa system using a baseline of 150 feet would be at least ten times as accurate as a radar with a 15-foot antenna. A Mistram, using a baseline of 10,000 feet for position measurements, would be better than Azusa by a further factor of sixty, while its velocity data (from a baseline of 100,000 feet) would be 600 times better. Even after allowing for the foreshortening of the effective baseline (GDOP) for elevation measurements on targets removed from zenith, these improvement factors would have justified considerable effort and expense in an interferometer system. The reasons for the failure of interferometer systems to meet these expectations have been obscured by difficulties in establishing standards against which the errors can be measured, but the results of atmospheric studies provide clear evidence that the initial assumption as to the size of the tropospheric fluctuations was in error.

The Nature of Tropospheric Fluctuations

Refraction studies performed at the Central Radio Propagation Laboratory of the National Bureau of Standards had provided by early 1960 a description of the frequency spectrum of tropospheric fluctuations.[12] A summary of this data is shown in Figure 4, where the power spectral densities of range and refractivity variations for two paths are plotted on logarithmic scales covering several decades. The range fluctuation is in terms of parts per million of the total path length, which was between 10 and 15 miles at an elevation angle of about 6°. The most notable feature of the range spectrum is its steady increase in intensity with decreasing frequency, down to one cycle per day, and the absence of any overwhelming daily component. This suggests that the refractivity variations are not the result of changing temperature, but may reflect the drift of a more or less "rigid" pattern of anomalies across the measurement paths.[13] Later measurements of phase difference variations across baselines of several lengths tended to confirm the equivalence of the temporal and spatial spectra.[14] The top scale of Figure 4 has been added by this writer to the NBS plot, and used to derive the errors which would have been observed across the Hawaii baselines if the fluctuations had resulted entirely from atmospheric drift at 10 ft/sec. The results are compared in Figure 5 with the measured data, expressed in terms of angle error for different baseline lengths (for the 15-mi path at 6°).

The Hawaii baseline data was taken over limited periods of time, and hence could not show the low-frequency components of error which are predicted by the spectrum of Figure 4. In addition, since the study was intended to measure variations only above about 0.003 cps, special efforts were made to eliminate the longer-term "trends" in the data, by selection of samples and by fitting the data to smooth curves. These low-frequency components, however, are the errors which would produce the largest effects on long-baseline systems, and their inclusion leads to the solid curve on Figure 5. If they are removed by a filtering process from the spectrum of Figure 4, the calculated errors agree almost exactly with measured data from which "trends" were removed. The conclusion which has been drawn from the NBS baseline data[14], that errors will vary inversely with baseline length for baselines beyond about 3000 feet, is only true if we are willing to ignore the error spectrum below 0.003 cps. If these low-frequency errors are important to us (and we will show later that they are of great importance in tracking targets with appreciable angular rates), then we must consider that the error varies only as the inverse fourth root of baseline, for baselines less than 100,000 feet. When compared with antennas whose aperture is large (e.g. 30 to 100 feet), the reduction in error with baseline length will be negligible until this length reaches several hundred feet.

The spectrum of Figure 4 also provides a means of predicting the predominating frequencies of refractive error in baseline systems of any length. When we combine the fluctuation spectrum $W(f)$ with a filter function $G_b(f)$ representing the response of the baseline, as in Figure 6, we obtain the spectrum of the range-difference error across that baseline. The variance of the error component in a given octave or decade of frequency is proportional to the spectral density times the center frequency of the octave or decade.[15] Thus, the largest error component will appear near the frequency $f_b = 0.22\, b/v_w$, where b is baseline length and v_w is drift velocity across the measurement path. Substantial error components will spread over the frequency range from one-tenth to ten times this frequency. For the assumed 10 ft/sec wind velocity normal to the measurement path, the largest component will appear near 0.03 cps for b = 100 ft, 0.003 cps for b = 1000 ft, and 0.0003 cps for b = 10,000 ft. If the low-frequency components are removed by limiting the observation time t_1 or by intentional high-pass filtering, the error will appear largest at the low-frequency cut-off of this filter, if this is above f_b. This fact explains the observation of "cyclic" errors in such systems as Azusa.[16] As the observation period is increased, the magnitude of the error and the amplitude of the low-frequency component will grow larger, until all components down to about $f_b/10$ are observed or until the system is recalibrated often enough to suppress the additional low-frequency error. We should note that the low-frequency error components, although very hard to observe in many tests, can lead to significant errors in prediction of missile impacts or satellite orbits. The measured data can appear free from undesired "noise", and yet contain trends which degrade its accuracy in a more serious way (noise can be removed by data smoothing, while low-frequency error cannot). Verification of this effect is found in monthly reports of instrumentation accuracy, which over many periods have shown less than two-to-one advantage in impact prediction accuracy for Azusa over tracking radar, although the advantage in tracking accuracy is represented to be nearer ten to one. The difference lies, presumably, in the fact that much of the radar error is in that region of the spectrum where it may be recognized (and smoothed), while the major interferometer error remains hidden under the actual changes in missile position and velocity. When trajectories measured by different "precise" instruments are compared, the differences in position are frequently far greater than permitted by the confidence limits assigned to the measurements. Although calibration and operating procedure may be partially responsible, much of the spread in data must be attributed to unrecognized atmospheric fluctuation.

Correction of Low-Frequency Fluctuations

The subject of correction for fluctuating errors has been analysed quite thoroughly by Thayer,[17] and we will not repeat his discussion here. We may note, however, that the corrections are effective only in the frequency region below 10^{-4} cps, and hence that they provide significant advantages only for baselines longer than about 100,000 feet. Furthermore, it appears that any refractivity data gathered at stations within about 300,000 feet of each other should be averaged to obtain a single correction applicable to all stations within that region. Corrections for horizontal gradients of refractivity do not appear profitable unless a great deal of additional data is available over a range of altitudes up to about 10,000 feet.

Effects on Velocity Data

When the presence of low-frequency fluctuations became obvious, hopes for extreme positional accuracies in interferometer systems faded. It was still expected, however, that the velocity data specifications might be met, since the derivatives of these slow fluctuations would contribute little to the over-all error. The spectrum of the velocity error for a baseline system is found by multiplying the position error spectrum, Figure 6, by $(2\pi f)^2$. In the case of the 1000-foot system, the resulting spectrum appears as in Figure 7, with most of the error in the band from 0.01 to 1 cps. In this velocity spectrum, it makes little difference whether the very low frequencies are included or not, and the angle velocity error does indeed vary inversely with the baseline length (above b = 100 ft), as shown in Figure 8. However, the original fluctuation spectrum, Figure 4, applied only to a fixed path, and the frequencies of the fluctuations depended upon drift velocity of the atmosphere across this path. If we are attempting to measure velocity, we must assume that the measurement paths follow the target, and move through the atmosphere with a velocity which depends upon the angular rate of the target, or its range and tangential velocity. To the extent that the original spectrum could be attributed to drift of a rigid pattern of refractivity across the path, we must now modify its frequency and amplitude scales to account for the relative velocity of moving paths in the atmosphere. The new relative velocity may range from 10 to 100 times the wind drift velocity (in Norton's recent paper[15] he gives the factor as 50 to 500). The position error remains the same, as the increase in frequency scale is matched by a decrease in spectral density. However, the velocity error spectrum contains also the factor $(2\pi f)^2$, and this increases the rms velocity error in direct proportion to the relative velocity. This increase is shown in the upper curves of Figure 8, which apply to a typical satellite track at long range, where the midpoint of the tropospheric path moves at 250 ft/sec. The additional error will not be apparent under controlled test conditions on fixed targets, nor will it appear when a missile is tracked on a path at constant angle, as in some guidance applications. It will be present, however, whenever the paths move through the troposphere at velocities greater than the normal wind drift speed. The new error components will be at higher frequencies, and therefore may be smoothed more effectively in data filters of limited lag time. Curves are shown for smoothing times of two and 20 seconds (lags of one and 10 sec). The effect of smoothing is to equalize the error for all systems whose baseline (or aperture) is less than a certain length. Systems of longer baseline will exhibit errors whose period exceeds the smoothing time, and whose magnitude varies inversely with baseline length.

Ionospheric Errors

The refractivity of the ionosphere is subject to a much greater range of variation than that of the troposphere. In addition to the systematic variation in refractivity as a function of frequency, for a given electron density, there are large variations in electron density with altitude, time of day, time of the sunspot cycle, latitude, and magnetic field conditions. For this reason, attempts to apply corrections for ionospheric errors have met with very limited success. Systems operating at VHF with two well-separated frequencies have been able to obtain corrected data by direct comparison of the two measurements,[18] but the use of ionospheric soundings has provided correction to no better than 50% of the initial error levels, and the same or better accuracy can be achieved by reference to monthly predictions of electron density.[9] Relationships between refractivity, operating frequency, and critical frequency are shown in Figure 9, which indicates the approximate bounds of system operation for precise tracking. Average values of range and angle error for daytime conditions are shown in Figures 10 and 11. Short-term error fluctuations (measured over periods of a few minutes) can be expected to have rms values near 1% of the average error, for distant targets, and perhaps 2-3% for targets within the lower ionosphere. Larger values are to be expected during periods of magnetic disturbance.

Because of the variability of ionospheric error, the preferred solution to precise tracking problems lies in the use of frequencies high enough to bring the average error below the tolerance level. Usually, this implies operation in the band 3000 - 10,000 mc, although tracking to the order of 30 feet is possible as low as 2000 mc. If data is available for correction, it may reduce the error somewhat under "normal" conditions, but the increase in error during disturbances will then lead to a greater spread in system performance levels.

Conclusions

The development of systems and procedures for reduction of atmospheric error has been reviewed, and some of the pitfalls in error estimation have been pointed out. In general, it can be stated that the statistical results of radio meteorological studies have kept pace with the needs of instrumentation system designers and users, but that the demands of the users have gone beyond the level to which the atmospheric errors can be predicted on a deterministic basis. Ignoring some of the important error components caused by long-period tropospheric changes in the measurement path, we have produced instruments which can measure to a precision ten to one hundred times beyond the normal variations in the wavefronts arriving at the antennas, and have neglected the geographical spreading of instrument sites which offers the only solution to precise tracking requirements. In the process, we have learned a great deal about atmospheric variations, but most of this knowledge could have been obtained more economically from experiments of the type performed by the National Bureau of Standards. The problems caused by atmospheric errors is not as bad as might be thought from the previous discussion, because the accuracies of existing instruments are, in fact, adequate for most purposes. Errors which exceed the expectations by factors of ten to one hundred are often not noticed, since there are no standards against which these errors can be measured under actual operating conditions. In the future, however, we can hope that the constraints imposed by the atmosphere on tracking accuracy will be recognized before development is carried out, and that maximum advantage will be taken of those factors which will reduce error in an economical way. Furthermore, we should note that real improvements in system accuracy are unlikely to be made until the actual limits of existing instruments are appreciated by the users. A major step in this direction will have been taken when quality analysis procedures are devised to cover all regions of the error spectrum, rather than the common subdivisions of "noise" and "bias". The mathematical attractiveness of the random-noise plus fixed-bias model is undisputed, but its effectiveness in predicting errors (and impact points) is open to serious question. This is probably the largest single factor producing the "missile accuracy gap" which separates the boundary of the confidence limit of measured data from the actual position or velocity of the target.

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TABLE I

Optimum Accuracy of Range and Angle Corrections

Long-Range Case (R > 300 n. mi.)		$\theta_o = 5^o$	$\theta_o = 20^o$
Initial range bias, ΔR_e	(ft)	75	22
Residual range bias, σ_{rb}	(ft)	0.75	0.2
% residual error		1	1
Initial angle bias, δ^t	(μrad)	3500	900
Residual angle bias, $\sigma_{\theta b}$	(μrad)	70	20
% residual error		2	2
Short-Range Case (R = 50 n. mi.)			
Initial range bias, ΔR_e	(ft)	22	7
Residual range bias, σ_{rb}	(ft)	0.5	0.15
% residual error		2	2
Initial angle bias, δ^t	(μrad)	2000	700
Residual angle bias, $\sigma_{\theta b}$	(μrad)	60	20
% residual error		3	3

(Values shown should be doubled for disturbed meteorological conditions such as heavy cloud cover, fronts, and inversions; also for lack of reliable and frequent soundings covering the entire tropospheric path used in measurement).

(from Ref. 9)

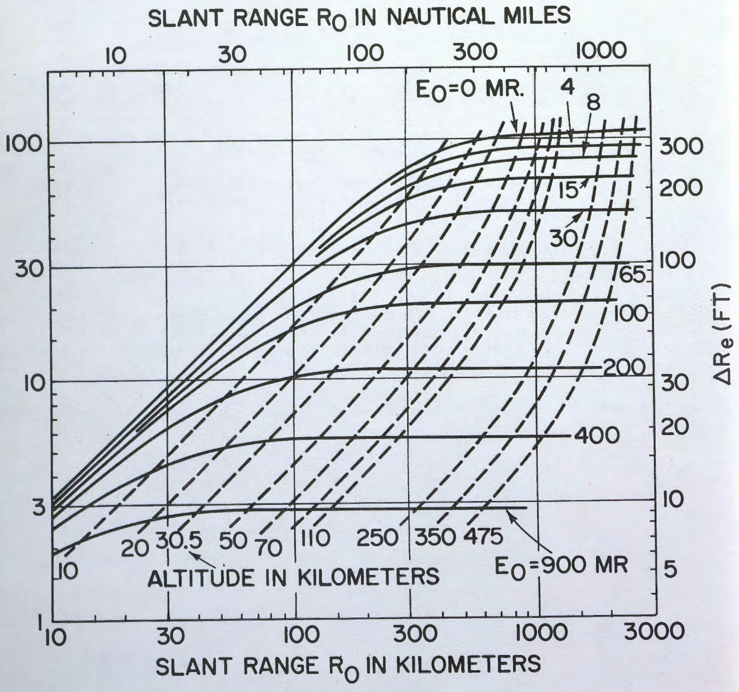

Figure 1. Range bias vs. range for CRPL exponential reference atmosphere, $N_o = 313\ \mu$ (from Ref. 9).

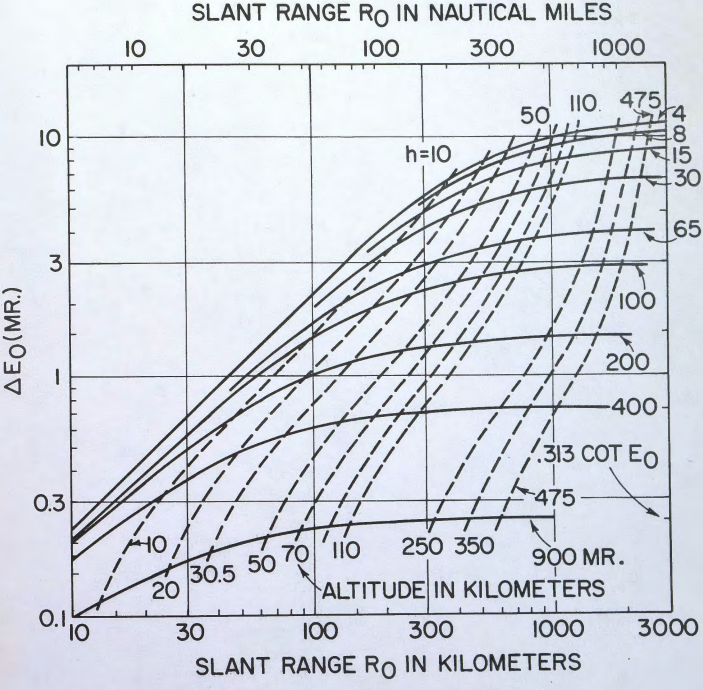

Figure 2. Tracker elevation angle error vs. range for CRPL exponential reference atmosphere, N_o = 313 (from Ref. 9).

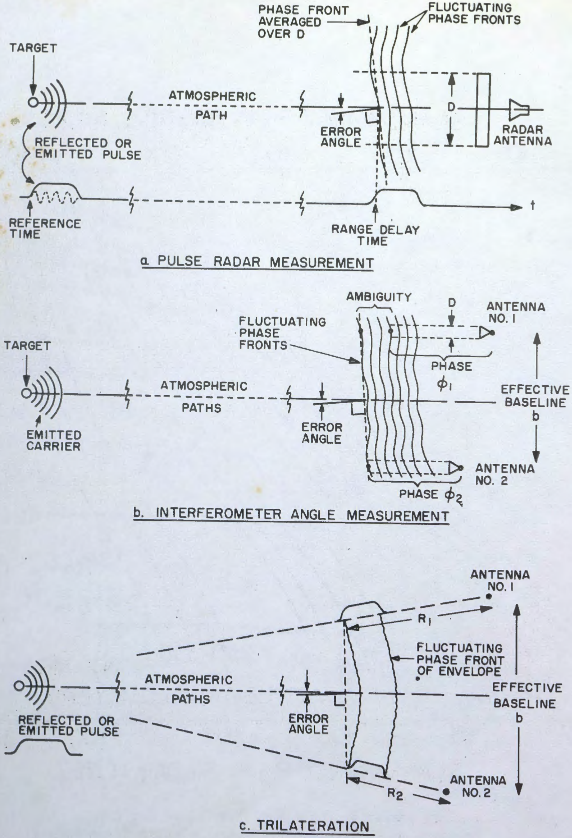

Figure 3. Fluctuation effects on different instruments.

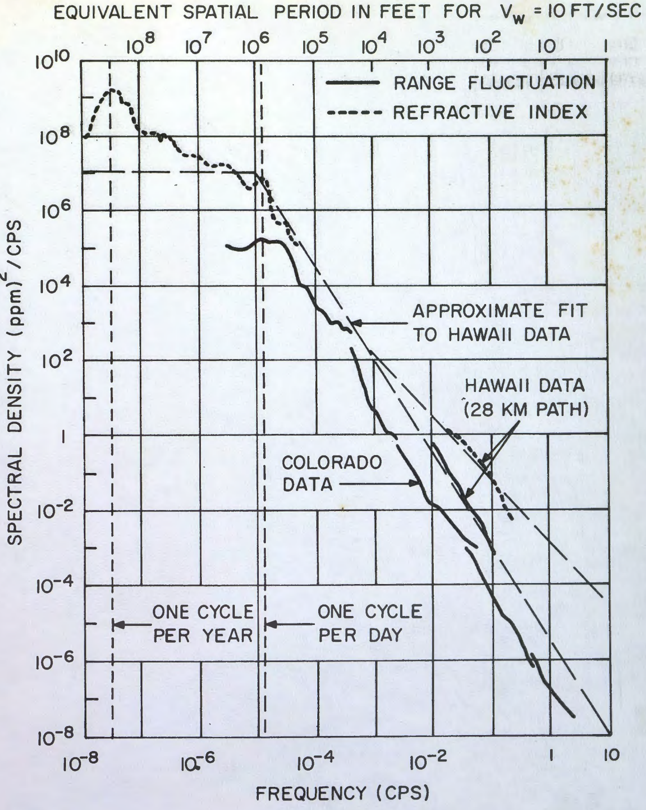

Figure 4. Spectra of refractivity and range fluctuation (NBS data, Ref. 12).

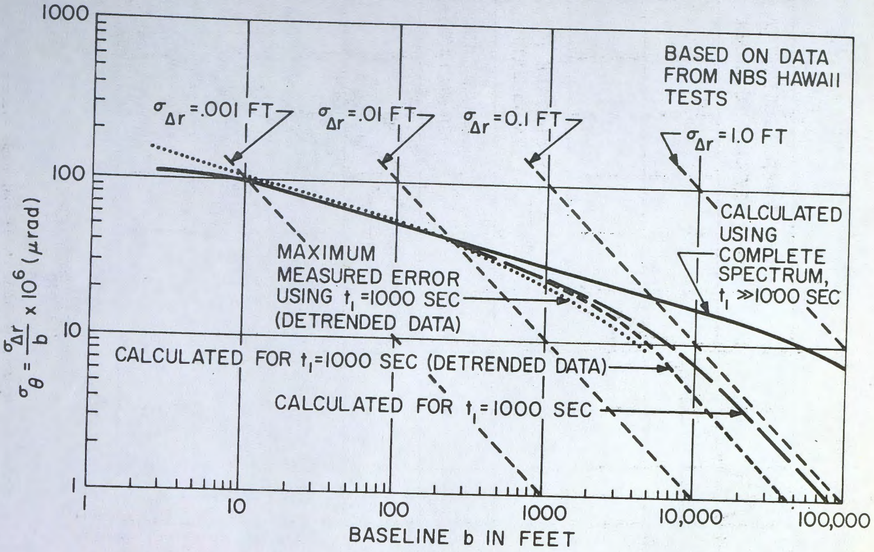

Figure 5. Equivalent angular position error σ_θ vs. baseline length b, for complete spectrum and for limited observation time t_1 (from Ref. 13).

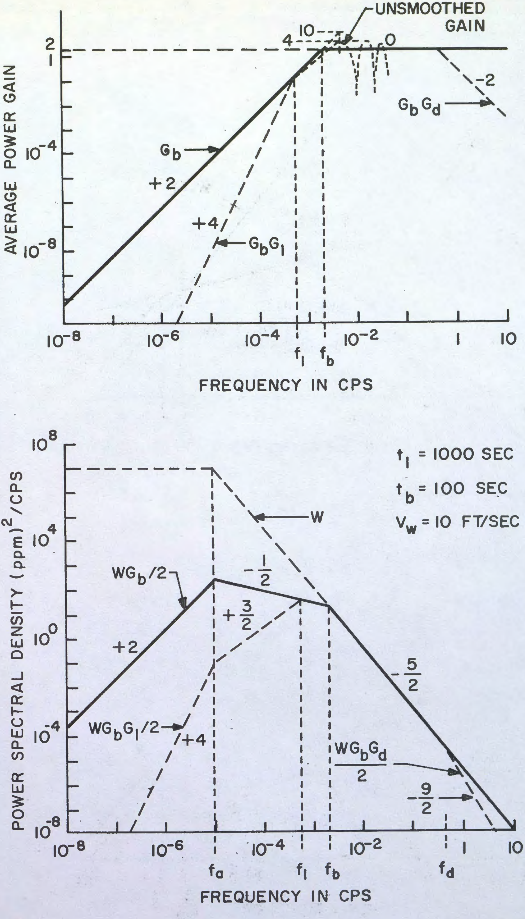

Figure 6. Response function of baseline system, G_b, and resulting range-difference fluctuation spectrum, WG_b, for b = 1000 ft, wind speed 10 ft/sec. (from Ref. 13).

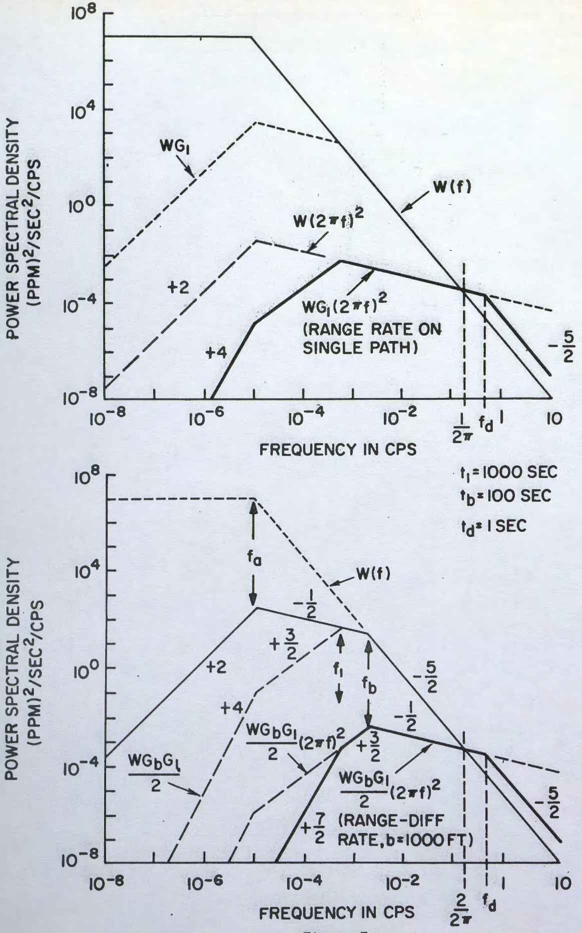

Figure 7.

Velocity and velocity-difference fluctuation spectra, for b = 1000 ft, wind speed 10 ft/sec. (from Ref. 13).

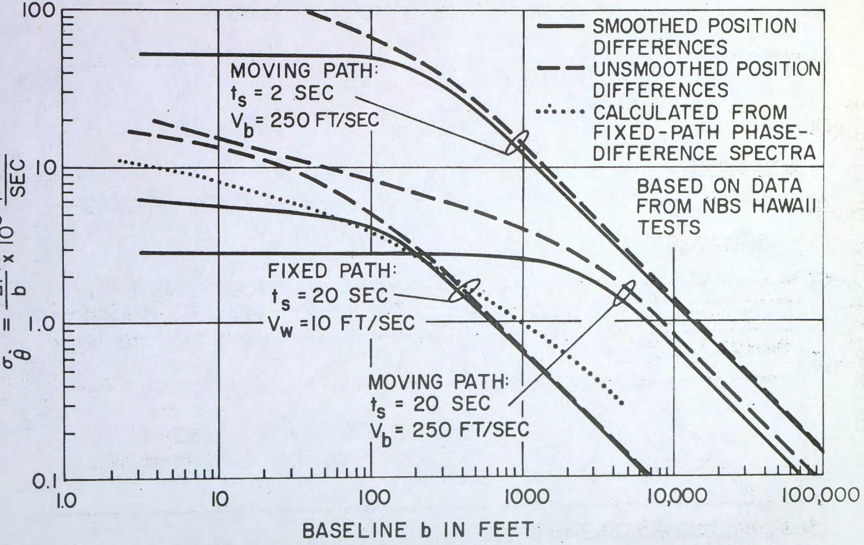

Figure 8. Equivalent angular rate error $\sigma_{\dot{\theta}}$ vs. baseline length b, for different relative beam velocities v_b and smoothing times t_s (from Ref. 13).

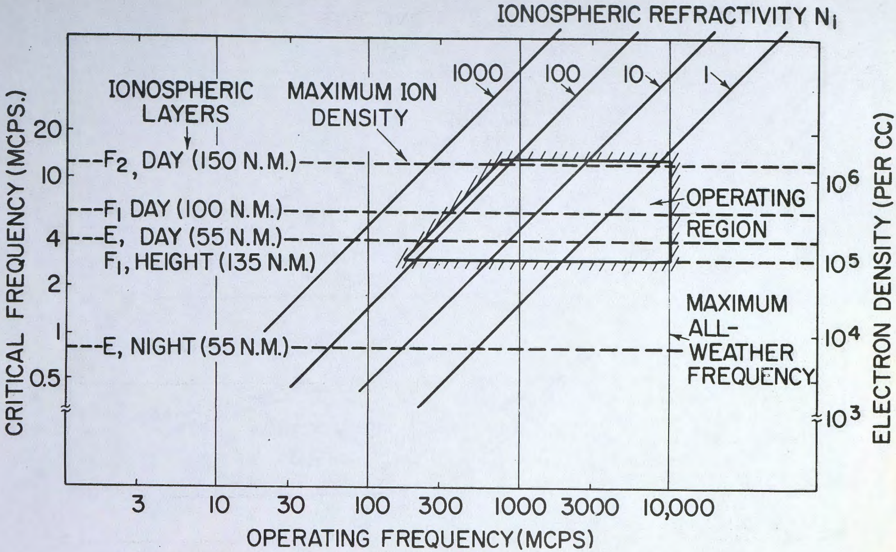

Figure 9. Relationship between ionospheric refractivity N_i, critical frequency, and operating frequency, showing limits for precise instrumentation.

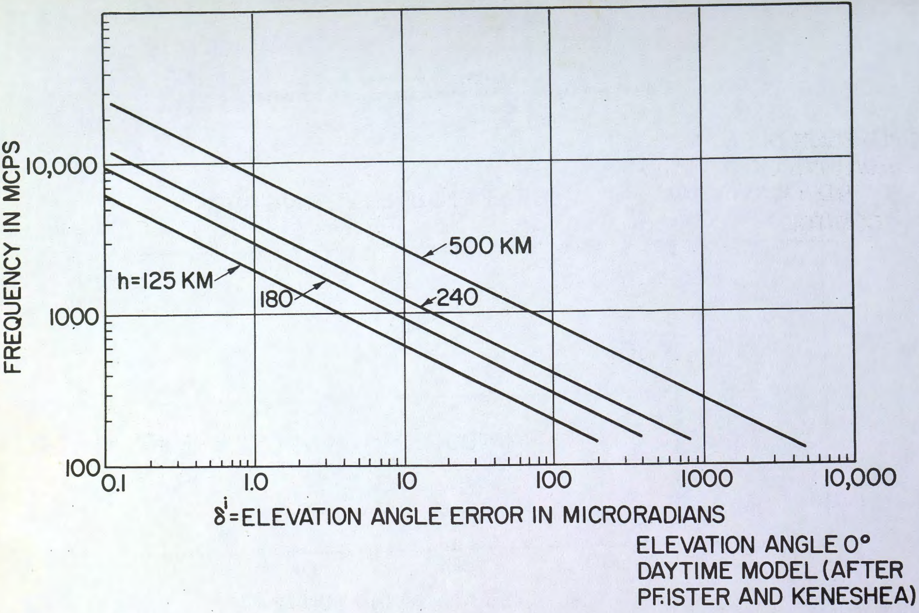

Figure 11. Ionospheric angle error vs. frequency (after Pfister and Keneshea, AFCRL).

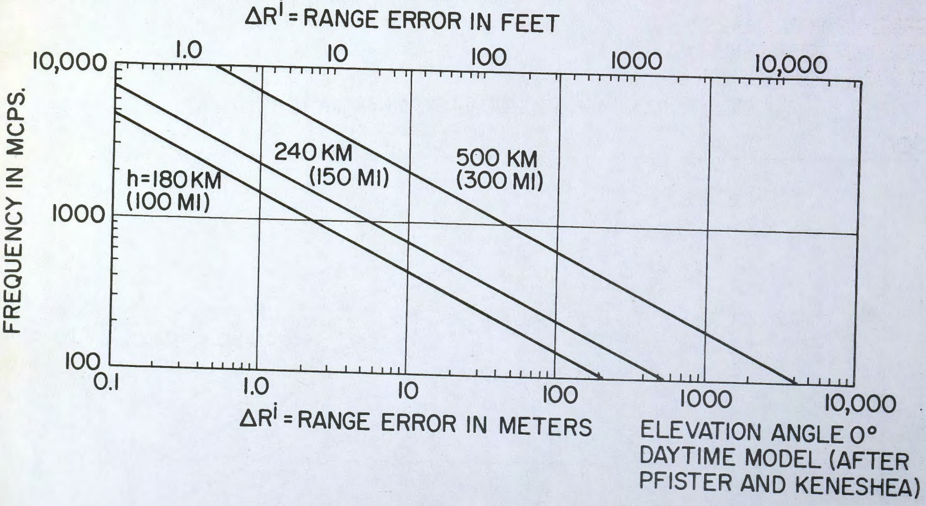

Figure 10. Ionospheric range error vs. frequency (after Pfister and Keneshea, AFCRL).